

Comment on "Extensions of the Falkner-Skan Similar Solutions to Flows with Surface Curvature"

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IT is disappointing and surprising to note that Murphy¹ repeats his previous mistake^{2,3} of making an order-of-magnitude analysis *before* differentiating his equations of motion. The errors so introduced have already been pointed out by the present writers⁴ in presenting what they believe to be a correct and more far-reaching investigation. A number of other algebraic errors have been taken over from Ref. 3 to Ref. 1, e.g., in Eqs. (28) and (29) of Ref. 1. Furthermore, Murphy's integrations are continued to $\eta = 10$ and to $\Omega = \pm 0.1$, i.e., to $\Omega\eta = Ky = \pm 1$, a value that is not negligible in comparison with 1 as he assumed in deriving his approximate equations. Thus, although his qualitative conclusions about the effect of surface curvature are correct, all of Murphy's detailed results, except those for zero curvature, are at fault.

References

- 1 Murphy, J. S., "Extensions of the Falkner-Skan similar solutions to flow with surface curvature," *AIAA J.* **3**, 2043-2049 (1965).
- 2 Murphy, J. S., "Some effects of surface curvature on laminar boundary-layer flow," *J. Aeronaut. Sci.* **20**, 338-344 (1953).
- 3 Murphy, J. S., "Extensions of the Falkner-Skan similar solutions of the boundary-layer equations to flows with surface curvature," Douglas Aircraft Co. Rept. ES 40487 (August 15, 1961).
- 4 Massey, B. S. and Clayton, B. R., "Laminar boundary layers and their separation from curved surfaces," *J. Basic Eng.* **87**, 483-494 (1965).

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Comments on "Transport Properties of Hydrogen"

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IN a recent paper, Brezing¹ has presented the results of his calculations of the transport properties of ionized hydrogen. In the light of research on transport properties of ionized gases performed here^{2,3} and elsewhere, several statements and conclusions in his article are open to question. In order of their appearance they are as follows:

- 1) It has been proved that charge transfer affects only the $Q^{(1)}$ cross sections for odd l .⁴ Hence, Eq. (9) for $Q^{(1)}$ is correct but $Q^{(2)}$ is not the same and must be computed taking into account only the elastic forces. The difference between these two cross sections can then be quite large. For argon $Q^{(1)}$ is typically $3 \times Q^{(2)}$.³
- 2) The thermal conductivity of binary mixtures is, in general, not proportional to the viscosity (remark just above

Eq. (13) and also in Sec. III under thermal conductivity). In the special case of the fully ionized gas where the viscosity is effectively that of the ions, the thermal flux is carried mainly by the electrons, and the only interaction is Coulombic, then these two coefficients will be proportional. But they are not necessarily proportional over a large range of partial ionization where different intermolecular forces determine the two coefficients, i.e., atom-ion and atom-atom forces for the viscosity and electron-ion, electron-atom and electron-electron forces for the thermal conductivity.

3) Equation (18) is incorrect. From the correct form⁵ we see that Eq. (18) is really the expression for the first approximation to D_{ei} ($\neq D_{ie}$) when atom and ion masses are equal, and that a compensating error in Eq. (19) results in the correct first approximation to the electrical conductivity. In the same section it is also implied that the second approximations to the multicomponent diffusion coefficients in the ternary mixture may be obtained by substituting the second approximation to the binary coefficients into Eq. (18) (after correction). Although this is probably true at very high or very low ionization, it has apparently not been proved in the intermediate case. The correct form of the second approximation to this coefficient has been given elsewhere.^{2,6}

4) The most serious discrepancy occurs with the thermal conductivity λ . Brezing reports that the λ which he computed with the expressions of Ref. 5 differed by not more than 12% from the value obtained with the expression of Spitzer and Härm.⁷ This conclusion disagrees with results from several sources. In what appears to be the earliest treatment of the properties of fully ionized gases, Landshoff^{8,9} found that accurate values of the thermal conductivity could be obtained only if three or more terms were retained in the polynomial expansion of the perturbation to the electron equilibrium distribution function.† If two terms were used, as is done in Ref. 5, then the computed thermal conductivity is only 0.42 of the value obtained by Spitzer and Härm. Although Landshoff neglected the contribution of the ion in his treatment, its inclusion for hydrogen increases the thermal conductivity by only a few percent.² It is evident that values of the thermal conductivity as computed with the expressions of Ref. 5 must be viewed with scepticism when ionization is more than slight.

If there are no numerical errors in the computations of the thermal conductivity, there are two possible reasons for the apparent agreement of λ with that for the fully ionized gas. One possibility, which has occurred in other investigations, is that Brezing does not report the true thermal conductivity. This coefficient is evaluated correctly only when all of the diffusion fluxes in the mixture have been set equal to zero. (See Ref. 5, p. 483.) In this way, one is led to the distinction between λ and λ' in Ref. 5. In the un-ionized gas λ' is within a few percent of λ so this distinction is not important. In fully ionized hydrogen, $\lambda' = 1.87\lambda$ in the second approximation,² so the distinction is very important. It does appear, however, that Brezing has taken this point into account.

Another possible source of the discrepancy is the Coulomb cross section. Several investigators have adjusted the numerical constants in this cross section to insure that the thermal conductivity takes on the correct value at full ionization. Eastlund¹⁰ has inserted constants in the expressions of Ref. 5 to accomplish the same purpose. Since Brezing does not actually quote the expressions for either the charged cross sections or for the thermal conductivity, it is possible that he has followed one of these two approaches. Since neither has a theoretical basis, it leaves the resultant values of this coefficient open to question.

5) Discrepancies occur also in the values reported for the binary atom-ion diffusion coefficients. Brezing reports a

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† Note, however, that he calls the coefficients obtained with two terms the first approximation. This nomenclature differs from that of Chap. 7 of Ref. 5 and of Ref. 2.